Blind Channel Estimation in Spatial Multiplexing Systems using Nonredundant Antenna Precoding*

Helmut Bölcskei[1)], Robert W. Heath Jr., and Arogyaswami J. Paulraj[2)]
Information Systems Laboratory, Stanford University
Packard 223, 350 Serra Mall, Stanford, CA 94305-9510
Phone: (650)-724-3640, Fax: (650)-723-8473, email: bolcskei@rascals.stanford.edu
[1)] on leave from the Dept. of Communications, Vienna University of Technology
[2)] on part-time leave at Gigabit Wireless Inc., 1916B Old Middlefield Way, Mountain View, CA 94043

Abstract

Wireless systems employing multiple antennas at the transmitter and the receiver have recently been shown to have the potential of achieving extraordinary bit rates. In this paper, we consider the problem of blind channel estimation in single-carrier broadband multi-antenna systems. Assuming a linear time-invariant matrix channel with delay spread, we propose an algorithm for the blind estimation of the matrix channel which uses second-order cyclostationary statistics. Our approach employs nonredundant precoding and yields unique estimates (up to a diagonal matrix of phase terms). Furthermore, it does not require knowledge of the channel order, imposes no restrictions on the channel zeros, and exhibits low sensitivity to stationary noise. We present simulation results demonstrating the performance of the proposed method.

1. Introduction

Deploying multiple antennas at both the transmitter and the receiver of a wireless system has recently been shown to yield extraordinary bit rates [1]-[4]. The corresponding technology, known as spatial multiplexing [1] or BLAST [2, 5], allows an impressive increase in data rate in a wireless radio link without additional power or bandwidth consumption. In practice, however, to get the promised increase, accurate channel state information is required in the receiver. This information can be obtained by sending training data and estimating the channel [6, 7]. The training overhead required, unfortunately, is more significant in estimating multiple-input multiple-output (MIMO) channels. To avoid this problem we propose a method for blind channel estimation.

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Blind estimation of a time-dispersive space-time channel is a practical example of the need for blind estimation of a general MIMO system. There is an abundance of literature on that subject. Due to lack of space, we shall not discuss all the existing ideas and algorithms; rather we refer the reader to [8] which contains an excellent overview of the subject and an extensive reference list. Some recent references are for example [9, 10, 11].

In this paper, we propose a nonredundant precoding scheme, which allows the receiver to blindly estimate the MIMO system using second-order cyclostationary statistics only. The new method identifies the matrix channel on a subchannel by subchannel basis, i.e., each scalar subchannel is identified individually. Important aspects of the proposed algorithm include: it does *not require knowledge of the channel order*, it does *not impose restrictions on channel zeros*, and *it exhibits low sensitivity* to stationary noise. Furthermore, our method yields unique estimates up to a diagonal matrix of phase terms. We note that an approach similar to the one proposed in this paper recently appeared in [12]. Our algorithm differs from that suggested in [12] in that we use cyclostationarity instead of conjugate cyclostationarity, it works for arbitrary symbol constellations and arbitrary stationary noise, the estimator performance does not degrade significantly if the number of sources increases, the knowledge of the pulse shaping filter can be incorporated to improve estimator performance, and the phase ambiguity is resolved up to only a diagonal matrix of phase terms. In this work, we restrict our attention to single-carrier modulation. An extension of the proposed idea to blind equalization of OFDM-based spatial multiplexing systems will be provided in [13].

The rest of this paper is organized as follows. In Section 2 broadband single-carrier based spatial multiplexing

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systems are described and our assumptions and some notation are introduced. In Section 3 we provide the novel algorithm and we consider a simple example with two transmit and two receive antennas. In Section 4 we describe generalizations of our algorithm to an arbitrary number of antennas, and we provide a refinement which allows to exploit knowledge of the transmitter pulse shaping filter to improve the estimator performance. Section 5 contains simulation results, and Section 6 provides our conclusions.

2 Spatial multiplexing systems

Let us briefly describe the system model used in this paper (see Fig. 1). In the following M_T and M_R denote the number of transmit-and receive antennas, respectively. Using single-carrier modulation, M_T different data streams are sent simultaneously from the M_T transmit antennas, pass through a time-dispersive matrix channel, and arrive at the M_R receive antennas. Since the individual transmit signals occupy the same frequency-band there is severe cochannel interference. This cochannel interference and channel dispersion can be mitigated in the receiver, for example, by employing a space-time equalizer [9].

We shall adopt the following notation. Let T denote the symbol period, $c_k^{(l)}$ the symbol stream transmitted from the l-th antenna, $d_{k,l}(t)$ the scalar subchannel between the k-th receive and the l-th transmit antenna, $\rho_k(t)$ the spatially uncorrelated stationary noise process for the k-th receive antenna, and $g(t)$ the transmitter pulse shaping filter assumed to be the same for all antennas. Then by defining the transmitted signal corresponding to the l-th antenna as

$$s_l(t) = \sum_{k=-\infty}^{\infty} c_k^{(l)} g(t - kT), \quad l = 0, 1, ..., M_T - 1,$$

the received signal at the k-th antenna can be written as

$$r_k(t) = \sum_{l=0}^{M_T-1} \int_{-\infty}^{\infty} d_{k,l}(t-\tau)\, s_l(\tau)\, d\tau + \rho_k(t).$$

Now let

$$\mathbf{r}(t) = [r_0(t)\ r_1(t)\ \dots\ r_{M_R-1}(t)]^T$$
$$\mathbf{s}(t) = [s_0(t)\ s_1(t)\ \dots\ s_{M_T-1}(t)]^T$$
$$\rho(t) = [\rho_0(t)\ \rho_1(t)\ \dots\ \rho_{M_R-1}(t)]^T$$
$$[\mathbf{D}(t)]_{k,l} = d_{k,l}(t),$$

where $k = 0, 1, ..., M_R - 1$ and $l = 0, 1, ..., M_T - 1$. Then we can write the MIMO system input/output relation as

$$\mathbf{r}(t) = (\mathbf{D} * \mathbf{s})(t) + \rho(t),$$

with the symbol $*$ standing for vector-matrix convolution. Note that each antenna k receives a superposition of the data symbols from each transmit antenna l convolved with the respective ISI channel $d_{k,l}(t)$ making both channel estimation and equalization quite challenging.

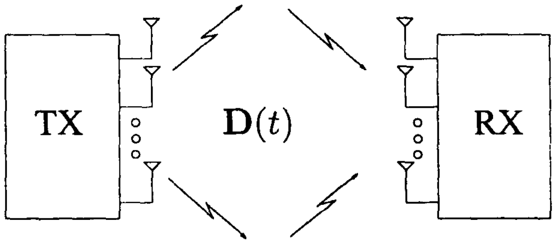

Fig. 1. Space-time coding modem.

3 Blind MIMO channel estimation

In this section, we shall introduce the novel algorithm for blind MIMO channel estimation and demonstrate the basic idea using a simple example with two transmit and two receive antennas. The generalization to more antennas and further refinements of the algorithm will be discussed in Sec. 4.

The basic idea of our algorithm is to perform nonredundant precoding on each transmitted sequence such that the cyclostationary statistics in the receiver allow separate identification of the scalar subchannels $d_{k,l}(t)$ ($k = 0, 1, ..., M_R - 1, l = 0, 1, ..., M_T - 1$). This is achieved by providing each transmit antenna with a different signature in the cyclostationary domain which allows us to null out all but one (or several) transmit antennas at a time. Using this approach, we can identify the matrix channel up to a constant diagonal matrix of phase factors, which is an ambiguity generally accepted in practice. Effectively, after space-time equalization, the symbol streams will be decoded up to a phase rotation, which will in general be different for different symbol streams. This ambiguity can either be resolved using higher-order statistics or by sending pilot symbols. Alternatively, if differential detection is employed, the phase rotation can be ignored.

Nonredundant precoding. Our approach is based on nonredundant precoding which consists of multiplying the transmitted data symbols (taken from a complex finite alphabet) by M-periodic precoding sequences. The specific criterion for designing the precoding sequences will be discussed later. We note that this form of precoding has previously been suggested by Serpedin and Giannakis in [14] to introduce cyclostationarity in the transmit signal thereby making blind channel estimation in single-antenna symbol-rate sampled single-carrier systems possible. The general idea of transmitter induced cyclostationarity has been suggested previously in [15, 16].

The precoded transmit signal corresponding to the l-th antenna is given by

$$s_l(t) = \sum_{k=-\infty}^{\infty} c_k^{(l)} a_k^{(l)} g(t-kT), \quad l = 0, 1, ..., M_T - 1,$$

where $a_k^{(l)} = a_{k+M}^{(l)}$ is the l-th M-periodic precoding sequence. This form of precoding evidently comes at the cost of slightly reduced spectral efficiency since the symbol constellation will be deformed due to the multiplication by the precoding constants.

Oversampling in the receiver. The receiver oversamples the vector signal $\mathbf{r}(t)$ by a factor of P (with respect to the symbol rate), i.e., it computes

$$\mathbf{r}[n] = \sum_{l=-\infty}^{\infty} \mathbf{H}[n-lP] \begin{bmatrix} c_l^{(0)} a_l^{(0)} \\ c_l^{(1)} a_l^{(1)} \\ \vdots \\ c_l^{(M_T-1)} a_l^{(M_T-1)} \end{bmatrix} + \rho[n]$$

with the vector $\mathbf{r}[n] \triangleq \mathbf{r}\left(n\frac{T}{P}\right)$ and the matrix function $\mathbf{H}[n] \triangleq \mathbf{H}\left(n\frac{T}{P}\right)$, where $\mathbf{H}(t)$ is obtained by convolving each of the entries of the matrix $\mathbf{D}(t)$ with the transmitter pulse shaping filter $g(t)$.

Cyclostationary statistics. Now, defining the correlation matrix of the vector random process $\mathbf{r}[n]$ as[1]

$$\mathbf{c}_r[n,\tau] = \mathcal{E}\{\mathbf{r}[n]\mathbf{r}^H[n-\tau]\}$$

and assuming that the data sequences $c_k^{(l)}$ are white and uncorrelated, i.e.,

$$\mathcal{E}\{c_k^{(l)} c_{k'}^{(l')^*}\} = \sigma_l^2 \delta[k-k']\delta[l-l']$$

and statistically independent of the data symbols, we obtain

$$\begin{aligned} \mathbf{c}_r[n,\tau] &= \sum_{l=-\infty}^{\infty} \mathbf{H}[n-lP] \mathrm{diag}\{|a_l^{(i)}|^2 \sigma_i^2\}_{i=0}^{M_T-1} \\ &\quad \mathbf{H}^H[n-lP-\tau] + \mathbf{c}_\rho[\tau], \end{aligned}$$

where $\mathbf{c}_\rho[\tau] = \mathcal{E}\{\rho[n]\,\rho^H[n-\tau]\}$. Using the M-periodicity of the precoding sequences, it follows that

$$\mathbf{c}_r[n,\tau] = \mathbf{c}_r[n+PM,\tau],$$

which implies that the vector random process $\mathbf{r}[n]$ is cyclostationary with cyclostationarity period PM. By cyclostationary vector random process with period PM we mean that each of the entries in the vector is a scalar cyclostationary random process with cyclostationarity period PM. Note that even if symbol rate sampling is employed (i.e. $P = 1$), cyclostationarity will be introduced in the transmitter thanks to the precoding operation, and the cyclostationarity period will be M. Therefore, oversampling is not crucial to our algorithm.

Next, we expand $\mathbf{c}_r[n,\tau]$ into a Fourier series with respect to n

$$\mathbf{C}_r[k,\tau] = \frac{1}{PM} \sum_{n=0}^{PM-1} \mathbf{c}_r[n,\tau]\, e^{-j\frac{2\pi}{PM}kn}$$

and we compute the z-transform of the resulting Fourier series coefficients to finally obtain the cyclic spectrum

$$\begin{aligned} \mathbf{S}_r[k,z] &= \sum_{\tau=-\infty}^{\infty} \mathbf{C}_r[k,\tau]\, z^{-\tau} \\ &= \frac{1}{PM} \mathbf{H}\left(z e^{j\frac{2\pi}{PM}k}\right) \mathrm{diag}\{\sigma_i^2 \Phi^{(i)}[k]\}_{i=0}^{M_T-1} \\ &\quad \tilde{\mathbf{H}}(z) + \mathbf{S}_\rho(z)\delta[k], \end{aligned} \quad (1)$$

where

$$\Phi^{(i)}[k] = \sum_{l=0}^{M-1} |a_l^{(i)}|^2 e^{-j\frac{2\pi}{M}kl},$$

$\tilde{\mathbf{H}}(z) = \mathbf{H}^H\left(\frac{1}{z^*}\right)$, and $\mathbf{S}_\rho(z) = \sum_{n=-\infty}^{\infty} \mathbf{c}_\rho[n]\, z^{-n}$. Note that the influence of stationary noise can be eliminated by considering nonzero cycles $k \neq 0$.

Simple example. We shall next explain the basic idea of our algorithm using a simple example with 2 transmit and 2 receive antennas. The more general case will briefly be discussed in Sec. 4. Consider the 4-periodic precoding sequences $a^{(0)} = [1 \;\; 1.1 \;\; 1 \;\; 1.1]$ and $a^{(1)} = [1.1 \;\; 1 \;\; 1 \;\; 1.1]$. This yields $\Phi^{(0)} = [4.42 \;\; 0 \;\; -0.42 \;\; 0]$ and $\Phi^{(1)} = [4.42 \;\; 0.21 + j0.21 \;\; 0 \;\; 0.21 - j0.21]$. In the following, in order to keep the discussion more general we shall stick to the notation M for the period of the precoding sequences instead of specializing to $M = 4$. We shall also use $k_1 = 1$ and $k_2 = 2$. Since $\Phi^{(0)}[\pm k_1] = 0$, we obtain from (1)

$$\begin{aligned} [\mathbf{S}_r[\pm k_1, z)]_{0,0} &= \frac{1}{PM} H_{0,1}\left(z e^{\pm j\frac{2\pi}{PM}k_1}\right) \\ &\quad \sigma_1^2 \Phi^{(1)}[\pm k_1] \tilde{H}_{0,1}(z) \end{aligned} \quad (2)$$

$$\begin{aligned} [\mathbf{S}_r[\pm k_1, z)]_{1,1} &= \frac{1}{PM} H_{1,1}\left(z e^{\pm j\frac{2\pi}{PM}k_1}\right) \\ &\quad \sigma_1^2 \Phi^{(1)}[\pm k_1] \tilde{H}_{1,1}(z). \end{aligned} \quad (3)$$

Similarly, since $\Phi^{(1)}[\pm k_2] = 0$, we get

$$\begin{aligned} [\mathbf{S}_r[\pm k_2, z)]_{0,0} &= \frac{1}{PM} H_{0,0}\left(z e^{\pm j\frac{2\pi}{PM}k_2}\right) \\ &\quad \sigma_0^2 \Phi^{(0)}[\pm k_2] \tilde{H}_{0,0}(z) \end{aligned} \quad (4)$$

[1] $\mathcal{E}$ stands for the expectation operator and the superscript ^{H} denotes conjugate transposition.

$$[\mathbf{S}_r[\pm k_2, z)]_{1,1} = \frac{1}{PM} H_{1,0}\left(z e^{\pm j \frac{2\pi}{PM} k_2}\right) \sigma_0^2 \Phi^{(0)}[\pm k_2] \tilde{H}_{1,0}(z). \quad (5)$$

Now, we can use a slightly modified version of the frequency-domain algorithm proposed by Tong et. al. in [17] for blind channel estimation in single-antenna single-carrier systems, to identify the subchannels $H_{0,1}(z)$ and $H_{1,1}(z)$ from $[\mathbf{S}_r[\pm k_1, z)]_{0,0}$ and $[\mathbf{S}_r[\pm k_1, z)]_{1,1}$, respectively, and the subchannels $H_{0,0}(z)$ and $H_{1,0}(z)$ from $[\mathbf{S}_r[\pm k_2, z)]_{0,0}$ and $[\mathbf{S}_r[\pm k_2, z)]_{1,1}$, respectively. Let us briefly describe the approach. From (4) we get

$$\frac{[\mathbf{S}_r[k_2, z)]_{0,0}}{[\mathbf{S}_r[-k_2, z)]_{0,0}} = \frac{H_{0,0}(z e^{j \frac{2\pi}{PM} k_2})}{H_{0,0}(z e^{-j \frac{2\pi}{PM} k_2})}, \quad (6)$$

where we have used the fact that $\Phi^{(0)}[k_2] = \Phi^{(0)}[-k_2]$. Eq. (6) can now be rewritten as

$$[\mathbf{S}_r[k_2, z)]_{0,0} H_{0,0}(z e^{-j \frac{2\pi}{PM} k_2}) - [\mathbf{S}_r[-k_2, z)]_{0,0} H_{0,0}(z e^{j \frac{2\pi}{PM} k_2}) = 0. \quad (7)$$

Denoting the length of the subchannel filter $H_{0,0}(z)$ as $L_{h_{0,0}}$, rewriting (7) in the time-domain yields

$$\sum_{l=0}^{L_{h_{0,0}}-1} \left[[\mathbf{C}_r[k_2, n-l]]_{0,0} e^{j \frac{2\pi}{PM} k_2 l} - [\mathbf{C}_r[-k_2, n-l]]_{0,0} e^{-j \frac{2\pi}{PM} k_2 l} \right] h_{0,0}[l] = 0 \quad (8)$$

for $n \in \mathbb{Z}$. In order to solve for the channel $h_{0,0}[n]$ we rewrite (8) in vector-matrix form as

$$\underbrace{[\mathbf{T}_{0,0}^{(k_2)} \mathbf{D}^{-k_2} - \mathbf{T}_{0,0}^{(-k_2)} \mathbf{D}^{k_2}]}_{\mathbf{S}_{0,0}^{(k_2,-k_2)}} \mathbf{h}_{0,0} = \mathbf{0} \quad (9)$$

with the $(3L_{h_{0,0}} - 2) \times L_{h_{0,0}}$ Toeplitz matrices $\mathbf{T}_{0,0}^{(\pm k_2)}$ with first row

$$[[\mathbf{C}_r[\pm k_2, -L_{h_{0,0}} + 1]_{0,0} \ 0 \ \ldots \ 0]$$

and first column

$$[[\mathbf{C}_r[\pm k_2, -L_{h_{0,0}} + 1]_{0,0} \ \ldots \ [\mathbf{C}_r[\pm k_2, 2L_{h_{0,0}} - 2]]_{0,0} \ 0 \ \ldots \ 0],$$

the $L_{h_{0,0}} \times L_{h_{0,0}}$ diagonal matrix

$$\mathbf{D} = \mathrm{diag}\{e^{-j \frac{2\pi}{PM} l}\}_{l=0}^{L_{h_{0,0}}-1}$$

and the $L_{h_{0,0}} \times 1$ vector

$$\mathbf{h}_{0,0} = [h_{0,0}[0] \ h_{0,0}[1] \ \ldots \ h_{0,0}[L_{h_{0,0}-1} - 1]]^T.$$

From (9) it follows that the subchannel $h_{0,0}$ can be uniquely recovered if the matrix $\mathbf{S}_{0,0}^{(k_2,-k_2)}$ has nullity one. Using Theorem 1 in [14] it can be shown that this is the case if and only if there is no $l \in [1, L_{h_{0,0}} - 1]$ such that $e^{j \frac{2\pi}{PM} 2 k_2 l} = 1$. Identifiability irrespectively of channel zero locations is therefore guaranteed if $L_{h_{0,0}} \leq \frac{PM}{2k_2}$. In our case this condition reduces to $L_{h_{0,0}} \leq P$. In practice the oversampling factor P and the period of the precoding sequences M should be chosen to satisfy the identifiability condition.

If the true correlations $\mathbf{C}_r[k, \tau]$ are known the channel can be found as the unique null eigenvector of $\mathbf{S}_{0,0}^{(k_2,-k_2)^H} \mathbf{S}_{0,0}^{(k_2,-k_2)}$ and the channel order can be estimated as the minimum order $L_{h_{0,0}}$ for which the matrix $\mathbf{S}_{0,0}^{(k_2,-k_2)}$ has nullity one. In practice, however, $\mathbf{C}_r[k, \tau]$ has to be estimated from a finite data record $\{\mathbf{r}[n]\}_{n=0}^{L-1}$ of length L according to [18]

$$\widehat{\mathbf{C}}_r[k, \tau] = \frac{1}{L} \sum_{n=0}^{L-1} \mathbf{r}[n] \mathbf{r}^H[n-\tau] e^{-j \frac{2\pi}{PM} k n}.$$

The channel estimate is then obtained as

$$\widehat{\mathbf{h}}_{0,0} = \arg\min_{\mathbf{h}_{0,0} \neq \mathbf{0}} \|\widehat{\mathbf{S}}_{0,0}^{(k_2,-k_2)} \mathbf{h}_{0,0}\|^2,$$

where $\widehat{\mathbf{S}}_{0,0}^{(k_2,-k_2)}$ is an estimate of $\mathbf{S}_{0,0}^{(k_2,-k_2)}$ obtained by replacing $\mathbf{C}_r[k, \tau]$ in (1) by the estimates $\widehat{\mathbf{C}}_r[k, \tau]$.

Performing the same procedure as above for (2),(3), and (5) yields estimates of the remaining scalar subchannels. Note that we eliminated the influence of stationary noise since we considered nonzero cycles $k \neq 0$ only. Each of the scalar subchannels has now been identified up to a phase factor, which will in general be different for different subchannels. We shall next describe how this remaining phase ambiguity can be resolved up to a diagonal matrix of phase terms.

Resolving the phase ambiguity. Assume that the true filter has been identified up to a phase ambiguity denoted as $e^{j\phi_{k,l}}$ for the filter $H_{k,l}(z)$ or equivalently the true filter $H_{k,l}(z)$ is given by $H_{k,l}(z) = e^{j\phi_{k,l}} H_{k,l}^{(e)}(z)$, where $H_{k,l}^{(e)}(z)$ is the estimate of the filter $H_{k,l}(z)$ obtained using the subspace-based algorithm described above. Considering $[\mathbf{S}_r[k_2, z)]_{1,0}$ we obtain

$$[\mathbf{S}_r[k_2, z)]_{1,0} = H_{1,0}^{(e)}(z e^{j \frac{2\pi}{PM} k_2}) \sigma_0^2 \Phi^{(0)}[k_2] \tilde{H}_{0,0}^{(e)}(z) e^{j(\phi_{1,0} - \phi_{0,0})},$$

which allows us to estimate the phase difference $\phi_e = \phi_{1,0} - \phi_{0,0}$. More specifically, going back to the time-domain we obtain an estimate for ϕ_e as

$$\widehat{\phi_e} = \arg\left\{ \frac{[\widehat{\mathbf{C}}_r[k_2, n]]_{1,0}}{\sigma_0^2 \Phi^{(0)}[k_2] r_h^{(e)}[n]} \right\}, \qquad n \in \mathcal{I},$$

where $r_h^{(e)}[n] = \sum_l h_{1,0}^{(e)}[n+l]e^{-j\frac{2\pi}{PM}k_2(n+l)}h_{0,0}^{(e)^*}[l]$ and $\mathcal{I} := \{n | r_h^{(e)}[n] \neq 0\}$. In practice, it is recommended to average over the interval $\mathcal{I}$, i.e.,

$$\widehat{\phi_e} = \frac{1}{|\mathcal{I}|} \sum_{n \in \mathcal{I}} \arg \left\{ \frac{[\widehat{\mathbf{C}}_r[k_2, n]]_{1,0}}{\sigma_0^2 \Phi^{(0)}[k_2] r_h^{(e)}[n]} \right\}.$$

Further averaging can be done by considering $[\mathbf{S}_r[k_2, z)]_{0,1}$ and performing the same procedure. The phase difference $\phi_{0,1} - \phi_{1,1}$ can be estimated similarly from $\widehat{\mathbf{C}}_r[k_1, n]$. Now, since we have estimated the phase differences $\phi_{1,0} - \phi_{0,0}$ and $\phi_{0,1} - \phi_{1,1}$, we have identified the channel transfer matrix up to a diagonal matrix of phase terms, i.e., any matrix

$$\mathbf{G}(z) = \mathbf{H}(z) \operatorname{diag}\{e^{j\phi_{i,i}}\}_{i=0,1}$$

is also a valid solution of our algorithm. This shows that using a space-time equalizer the individual symbol streams can be recovered up to a phase rotation.

4 Extensions of the algorithm

In this section, we present a refinement of the new algorithm, which allows to improve the estimator performance if the receiver knows the transmitter pulse shaping filter $g(t)$. We also describe how the algorithm can be generalized to more than two antennas.

Exploiting knowledge of the transmitter pulse shaping filter. Continuing with the simple example discussed in the previous section, if the sampling rate is high enough to avoid aliasing, we have

$$H_{k,l}(z) = G(z) D_{k,l}(z), \qquad k, l = 0, 1,$$

where $G(z) = \sum_{n=-\infty}^{\infty} g\left(n\frac{T}{P}\right) z^{-n}$ and $D_{k,l}(z) = \sum_{n=-\infty}^{\infty} d_{k,l}[n]\, z^{-n}$. Now, similarly to (7) we can build up a linear system of equations whose solution yields the channel estimate (i.e. the channel without transmitter pulse shaping filter) as

$$[\mathbf{S}_r[k_2, z)]_{0,0}\, D_{0,0}(ze^{-j\frac{2\pi}{PM}k_2})\, G(ze^{-j\frac{2\pi}{PM}k_2}) -$$
$$[\mathbf{S}_r[-k_2, z)]_{0,0}\, D_{0,0}(ze^{j\frac{2\pi}{PM}k_2})\, G(ze^{j\frac{2\pi}{PM}k_2}) = 0.$$

Since $G(z)$ is known at the receiver an equation similar to (9) can be built up and $d_{0,0}[n]$ can be estimated. The details of this algorithm will be reported in the multi-carrier context in [13]. Using a different subspace-based algorithm, it has been observed previously in the single-antenna case in [19] that incorporating knowledge of the transmitter pulse shaping filter in the channel estimation procedure yields improved estimator performance. Although a proof is unavailable thus far, simulation results indicate that this improves estimation accuracy in our case as well.

Generalization to arbitrary number of antennas. Let us next discuss the extension of our algorithm to an arbitrary number of antennas. Basically, one has to construct M_T precoding sequences such that there is exactly one sequence $a_l^{(i_1)}$ which satisfies $\Phi^{(i_1)}[s] \neq 0$ for at least one cycle $s \in [0, M-1]$ and $\Phi^{(i)}[s] = 0$ for $i \neq i_1$. Using this cycle s the corresponding filters $H_{l,i_1}(z)$ $(l = 0, 1, ..., M_R - 1)$ can be identified. Next, picking a cycle $s \in [0, M-1]$ and an index $i_2 \neq i_1$ such that $\Phi^{(i_2)}[s] \neq 0$, $\Phi^{(i_1)}[s] \neq 0$, and $\Phi^{(i)}[s] = 0$ for $i \neq i_1, i_2$, we can identify the subchannel filters $H_{l,i_2}(z)$ $(l = 0, 1, ..., M_R - 1)$ by subtracting out the estimates of the filters $H_{l,i_1}(z)$ and applying the algorithm provided in the previous section. This procedure has to be performed for all $i \in [0, M_T - 1]$ and therefore yields a column by column identification of the channel matrix. We note that this approach is prone to error propagation. However, the conditions imposed on the precoding sequences are not as restrictive as if one would require that the $a_l^{(i)}$ are such that a subchannel by subchannel identification can be performed without having to eliminate the previous estimates by subtracting them out. We finally note that we do not have a general procedure for the design of precoding sequences. For small numbers of antennas (which is usually the case in practice) it is fairly easy to obtain precoding sequences. An example for the case of three transmit antennas is given by

$$\begin{aligned} a^{(0)} &= [\alpha\ \beta\ \alpha\ \beta\ \alpha\ \beta] \\ a^{(1)} &= [\beta\ \alpha\ \alpha\ \alpha\ \alpha\ \beta] \\ a^{(2)} &= [\beta\ \alpha\ \alpha\ \beta\ \alpha\ \alpha], \end{aligned}$$

where $\alpha \neq \beta \neq 0$ are arbitrary (complex) constants.

5 Simulation Results

We finally provide simulation results demonstrating the performance of our algorithm on a simple 2 by 2 matrix channel. We did not assume knowledge of the pulse shaping filter $g(t)$. The overall channel to be identified is given by[2]

$$\begin{aligned} \mathbf{h}_{0,0} &= [1\ \ -1\ \ 2] \\ \mathbf{h}_{0,1} &= [1\ \ 1\ \ 1.2] \\ \mathbf{h}_{1,0} &= [1.5\ \ -1\ \ 1.5] \\ \mathbf{h}_{1,1} &= [0.5\ \ -0.75\ \ 0.75] \end{aligned}$$

with the precoding sequences $a^{(0)} = [1.1\ 1\ 1\ 1.1\ 1\ 1]$ and $a^{(1)} = [1.1\ 1.1\ 1.1\ 1\ 1\ 1]$, oversampling factor $P = 1$, and packet size of 1200 i.i.d. 4-QAM symbols. We used the MSE and the average bias, both averaged over 2000 independent Monte Carlo trials, to evaluate the chan-

[2] In the simulations the channels have been normalized to have norm 1.

nel estimation error. Fig. 2 shows the MSE and the average bias (both averaged over all four subchannels) as a function of SNR $= 10\log_{10}\left(\frac{\sigma_0^2+\sigma_1^2}{2\sigma_\rho^2}\right)$, where σ_ρ^2 denotes the noise variance of each of the scalar white noise processes $\rho_0[n]$ and $\rho_1[n]$.

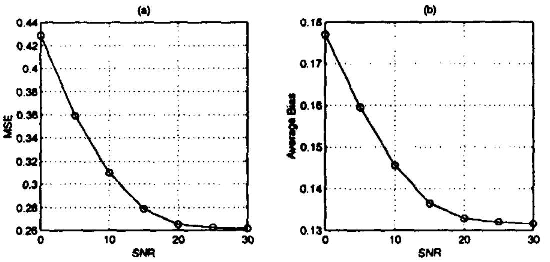

Fig. 2. Estimator performance: (a) MSE and (b) average bias as a function of SNR in dB.

We observe the intuitive result that the estimator performance improves with increasing SNR. Finally, Fig. 3 shows the estimator performance at an SNR of 15dB as a function of the data record length L used to estimate the cyclostationary statistics $\widehat{\mathbf{C}}_r[k,\tau]$.

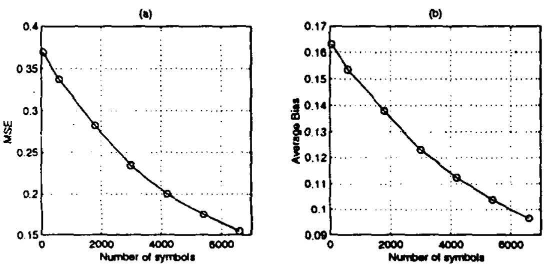

Fig. 3. Estimator performance as a function of data record length in symbols: (a) MSE and (b) average bias.

We can see that the performance of the estimator improves with increasing data record length. It has to be noted, however, that this simulation result also shows that rather long data record lengths are required to obtain good channel estimates.

6 Conclusion

We introduced an algorithm for blind MIMO channel estimation using second-order cyclostationary statistics. The novel method employs nonredundant precoding such that each transmit antenna is provided with a different signature in the cyclostationary domain which allows to null out all but one (or several) transmit antennas at a time. This makes a column by column identification of the matrix channel possible. The performance of our algorithm does not degrade significantly if the number of transmit antennas is increased, it does not require knowledge of the channel order, imposes no restrictions on channel zeros, and exhibits low sensitivity to stationary noise.

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